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A GRAPHIC METHOD FOR PRESENTING COMPARATIVE COST ANALYSES

By H. R. Bonner, United States Bureau of Education

Considerable demand has arisen for a method of unit cost analysis which will show graphically just how much money is being spent for various purposes and whether such expenditures are proportionately high or low. The following article presents such a method for analyzing and interpreting the expenditures incurred by nine state normal schools. The applicability of the plan outlined is not limited, however, to the educational field alone, but is equally adaptable to other activities where the expenditures of several plants or systems of any type are to be compared.

LIMITING THE STUDY

These nine state normal schools have been chosen for study from a list of 58 such schools submitting similar reports to the Bureau of Education for 1917–18, because they vary considerably in size and possess wide deviation from the general practice in the distribution of their costs among the generally recognized functions of expense. Only nine schools have been chosen, since they serve to illustrate this graphic method of presentation and the principles involved therein, and since it is not the purpose of this article to furnish a complete analysis of such statistics of all normal schools. The content of the subject has been sacrificed, therefore, for the sake of simplicity in presentation.

TABLE I—TOTAL CURRENT EXPENSES IN NINE STATE NORMAL SCHOOLS, 1917-18

Code number of school	Attend- ance weeks	Total current expenses	Cost of adminis- tration	Cost of instruction	Cost of operation of plant	Cost of mainte- nance	Miscella- neous costs
1	2	3	4	5	6	7	8
9 7 5 4 6 2 1 3 8	1,558 8,766 15,243 15,909 9,618 28,344 39,116 17,577 6,805	\$28,437 90,325 122,777 110,741 59,541 165,645 191,780 68,966 15,444	\$2,640 8,692 9,000 5,113 7,286 6,794 13,642 5,700 2,330	\$16,360 54,906 30,113 31,882 34,364 38,998 142,683 48,790 9,298	\$3,606 19,059 16,455 29,743 16,499 71,495 20,030 11,704 3,816	\$2,102 812 61,656 40,201 44,237 8,425 1,622	\$3,729 6,856 5,553 3,802 900 4,121 7,000 1,150
Totals	142,936	\$853,656	\$61,197	\$407,394	\$192,407	\$159,547	\$33,111
Average cost (per student per week)		\$5.97	\$ 0.43	\$ 2.85	\$ 1.35	\$1.11	\$0.23
Per cent of total			7.2	47.7	22.6	18.6	3.9

SIGNIFICANCE OF THE CODE NUMBER

Each school in Table I has been assigned a significant number which indicates the relative size of the institution as measured in attendance weeks. Thus, school No. 1 is the largest, having a total of 39,116 student or attendance weeks, and school No. 9 the smallest, with only 1,558 similar units. In ascertaining the number of attendance weeks, the attendance of one student for one week at the school is considered a unit. If a student attends for 36 weeks or one year, the institution is accredited with 36 student weeks or attendance weeks. If a student withdraws from school after attending for six weeks, the school is given credit for only six attendance weeks. Students in extension and correspondence courses have been omitted from consideration. Thus, each school is measured by the same unit, thereby permitting a just and fair comparison of expenditures.

ACCOUNTING TERMINOLOGY

In order that a comparison of expenditures may be justly made, it is necessary to eliminate from the field of study all unusual expenditures. Consequently, all capital outlays for new buildings, grounds, and new equipment have been excluded, and only those expenditures have been included which are likely to be constant from year to year and which are usually termed current expenses. It would be unfair to include capital outlays since possibly only one school in the nine would have spent any considerable sum for the construction of a new building during the year. Such unusual expenditures, which are designed to serve for a period of years to come, must be eliminated from consideration in such a study of annual expenditures. Consequently only current expenses are analyzed. This major classification includes the costs of administration, of instruction, of operation of plant, of maintenance, and of miscellaneous items. The cost of administration usually includes the salaries and expenses of the president or principal and of the controlling or governing board. The cost of instruction includes the salaries of deans and teachers, the cost of text-books and any amount spent for supplies (materials which when used are of no further value) used in instruction or in the actual teaching process. Under operation of plant are placed the wages of janitors, firemen, and engineers; the cost of fuel, water, light, and power; and the supplies used in keeping the school plant fit for occupancy. Maintenance costs include expenditures for repair work and for the replacement of outworn furniture, apparatus, etc. Miscellaneous costs constitute a catch-all group which includes the cost of medical inspection, transportation of pupils, school libraries, lunches, lecture courses, rent, insurance, contributions, and a host of other

auxiliary agencies. Great variation from the general practice, therefore, may be expected among the schools on this score. In Table I the total current expenses were classified and distributed by each school under these five subdivisions.

THE AVERAGE AND THE MIDDLE HALF

In this method of cost analysis, the arithmetical average is used as a basis with which to compare the total current expenses and also the various subdivisions of this total. It is found to possess a certain mathematical property not characteristic of the median, as will be shown below.* Even with so small a number of schools as nine, the median and the average are not materially different. Although the median is not used, its near relatives the first and third quartiles have been utilized to mark the boundaries beyond which it is questionable for an institution to transgress. A sort of hybrid has, therefore, been introduced—the middle half or "safety zone" and the average. These "safety" and "danger" zones have been indicated clearly in the accompanying figures on three different counts.

It has become almost an established practice, especially in the analysis of school costs, to divide the school systems studied into four equal or nearly equal groups, according to the magnitude of the unit cost. Schools having expenditures falling within the upper or lower fourths are usually considered as having high or low costs. The schools falling within the middle half of the array are within the so-called "safety zone." The extreme costs may be more or less justifiable if all the conditions were known, but ordinarily such justification does not exist. It might be contended that all high unit costs are commendable in view of the fact that such a meagre amount is often spent on education. It should also be borne in mind that small schools usually have comparatively high unit costs. A reduction in cost might mean the discontinuation of the institution.

THE AVERAGE COSTS

In Table I the average costs for the nine normal schools combined have been secured by dividing the respective total costs by the total number of attendance weeks. It is found that the average cost per attendance week for all purposes is \$5.97. This average implies that it costs \$5.97 to maintain a normal school for a week for each student in attendance. Of this amount, 43 cents goes for administration, \$2.85 for instruction, \$1.35 for the operation of plant, \$1.11 for maintenance costs; and 23 cents for miscellaneous purposes. Thus, 7.2

^{*}See Bulletin No. 81, 1919, Bureau of Education, by Blauch and Bonner.

per cent of the total cost in the "average" school goes for general control, 47.7 per cent for instruction, 22.6 per cent for operation, 18.6 per cent for maintenance, and 3.9 per cent for other purposes. These averages and percentages form the bases with which the practice of individual institutions is compared. Each school contributes its weight to the "make up" of these basal figures.

THE INDIVIDUAL COSTS

The average cost per attendance week has been computed for each institution and the results placed in Table II, in columns 2, 4, 9, 14, 19, and 24. The corresponding averages for all schools combined have been carried forward from the preceding table and placed in the proper columns in the same table. In school No. 9, the total cost per attendance week is \$18.25. Of this amount, \$1.69 is spent for administration, \$10.50 for instruction, \$2.31 for operation, \$1.35 for maintenance, and \$2.40 for miscellaneous purposes. These amounts are respectively 9.3, 57.6, 12.6, 7.4, and 13.1 per cents of the total, \$18.25. These proportional percentages are shown respectively in columns 8, 13, 18, 23, and 28 of the table.

THE TOTAL COST CURVE

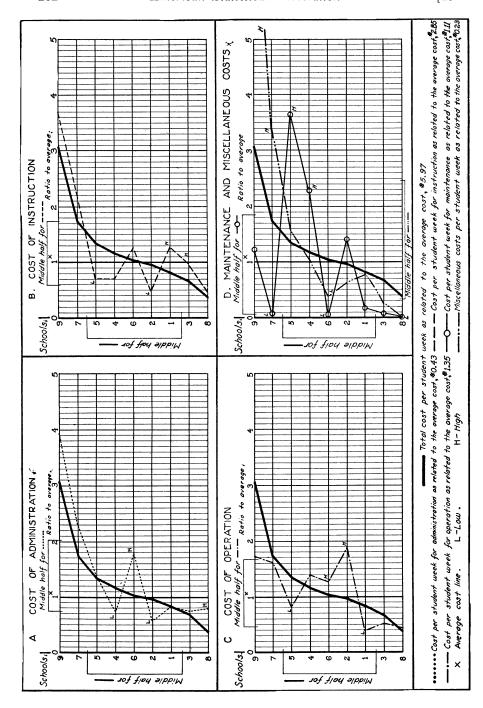
The costs in each institution and in all schools combined have been reduced to a uniform basis, viz., the cost per student week. The relationship of this weekly cost in each school is now compared with the average cost in all schools. The order in which the schools appear in both the tables and the graphs is determined by the magnitude of this cost per attendance week as shown in Table II, column 2, and by the continuous heavy curve in the diagrams. In school No. 9, the cost is \$18.25 or 3.06 times the average cost of \$5.97. In other words, the cost per student week in school No. 9 is over three times the average cost. This ratio is placed in Table II, column 3, and is used in locating the first point on the heavy curve in the charts. As this ratio indicates the deviation from the general practice geometrically, instead of arithmetically, it is best termed the ratio of geometrical deviation.

In two ways at least this ratio is superior to the arithmetical deviation so frequently found. For instance, the arithmetical deviation of the total cost in school No. 9 from the average cost is +\$12.28. The corresponding deviation of school No. 8 from this average is -\$3.70. The significance of these differences is not easily grasped. The first denotes that school No. 9 is higher than the average by \$12.28, and that school No. 8 is lower than the average by \$3.70. But neither shows what proportional reduction or increase is necessary to effect a com-

TABLE 11—AN ANALYSIS OF THE CURRENT EXPENSES PER STUDENT WEEK IN NINE STATE NORMAL SCHOOLS, 1917–18

, l	Per cent of		28	- 1	6 3.9	
Miscellaneous cost per "student week"	Geometrical deviation from total cost ratiof	Posi-	27	3.41 1.96 1.16 1.16	8 \$1.56	
		Neg- ative	26		*:38	
	Ratio to	age*	25	44.01 1.56.1 1.00 1.00 1.00 1.00 1.00	1.00	!
		3unom y	24	\$2.40 .78 .36 .24 .09 .14 .18	\$0.23	
L.	Per cent	total	23	7.4 20.2 36.3 36.3 26.7 2.3 2.3	18.6	
ance pe	metrical viation from tal cost	Posi- tive	22	2.70 1.95 1.43	\$1.69	
Cost of maintenance per "student week"	Geo Fo	Neg- ative	21		1.05.81.	
it of m	Ratio to	age*	50	1.22 .08 3.64 2.28 2.28 1.40 .05	1.00	ļ
ల		tanomA	19	12.6 \$1.35 21.1 0.09 13.4 4.04 22.8 0.53 22.8 0.05 43.2 1.56 10.4 21 17.1 0.09	22.6 \$1.11	
ınt	Per cent	total	18			
of pla week"	ometrical eviation from stal cost	Posi- tive	17	1.19	62 \$1.20	
Cost of operation of plant per "student week"	Geometrical deviation from total cost ratio.†	Neg- ative	16		++	
t of op er "st	Ratio to	age*	15	1.71 1.61 1.80 1.39 1.27 1.87 1.87 2.50	1.00	i
Cosi		tanomA	14	\$2.31 1.08 1.08 1.72 2.52 2.52 2.51 .51 .67	\$1.35	
	Per cent	total	13	24.6 60.8 24.6 28.7 7.7 7.7 7.1 60.4 60.4	47.7 \$1	
Cost of instruction per "student week"	Geometrical deviation from total cost ratio	Posi- tive	12	1.21 1.27 1.20 1.20 1.56 1.46 1.21	\$1.36	
nstruct nt wee		Neg- ative	Ξ		\$.56	
st of instructic student week	Ratio to	age*	10	3.69 2.20 2.20 7.70 1.25 1.25 1.28 1.28 1.28	1.00	:
పి`	JunomA			\$10.50 6.26 1.98 2.00 3.57 1.38 3.65 2.78	\$2.85	
ocr	Per cent	total	00	9.3 9.6 7.3 7.3 4.6 4.1 7.1 15.0	7.2	
Cost of administration per "student week"	trical tion m cost	Posi- tive	1	1.28 1.33 1.02 1.70 1.70 2.08	\$1.51	
	Geometrical deviation from total cost ratio†	Neg- ative	9		18.4	
	Ratio to	aver- age*	ro.	3.93 2.30 1.37 1.77 1.77 1.76 .81 .74	1.00	!
		tanomA.	4	\$1.69 .999 .32 .354 .354 .354	\$0.43	
Total current ex- penses per 'student week' Week' Ratio Ratio aver- Amount age*			ಣ	3.06 1.73 1.35 1.04 1.04 1.04 1.04 1.04 1.04 1.04 1.04	1.00	
Total current ex-	weel	tanomA		\$18.25 10.30 8.05 6.96 6.19 5.84 4.90 3.92	\$5.97	
	Code number of school		-	⊕25-64-621-88	Allschools	Legend

• Divide cost per student week by the average cost for all schools.
† Divide the ratios to the average cost for any function of expense by the corresponding ratio for the total cost in column 3, i. e., column 5 divided by column 3; column 3 for the average cost for any function of expense by the corresponding 3 for the triangle—point midway between the second and the third lowest deviations.
¶ Third quartile—point midway between the third and the second highest deviations.
¶ Third quartile—point midway between the third and the second highest deviations.



mendable practice. To arrive at this conception, a ratio of geometrical deviation must be resorted to, viz., divide \$12.28 by \$18.25 to ascertain the percentage of reduction necessary in school No. 9, or divide \$3.70 by \$2.27 to get the percentage of increase necessary for school No. 8 to approach the average. By reference to column 3 of the table, it is seen at once that the first school spends 3.06 times the average, and the second only .38 of the average. The points are clearly shown in the charts as the extreme points of the continuous heavy curve.

In further disapproval of the arithmetical deviation in studies of this type, it may be said that when it is used only one curve may be drawn on a diagram unless a double scale is used. In plotting curves of ratios of geometrical deviation, everything relates to the average, which becomes 100 per cent or 1 as shown in the diagrams.

In this study it is found that schools No. 9 and No. 7 lie in the highest and schools No. 8 and No. 3 in the lowest fourth, which are "danger zones." The more general practice is indicated for the other five schools bracketed at the extreme left of each diagram. The schools falling within the middle half and the relative variation of the total cost per student week in each school from the average cost for all schools, are readily apparent from the diagrams.

THE ADMINISTRATION COST CURVE

In a similar way the deviations of the costs for administration in the various schools from the average cost have been computed as shown in columns 4 and 5 of Table II and the corresponding ratios plotted in Figure 1, Part A. To read this curve, observe that school No. 9 spends 3.93 times as much per student week for administration as is spent on an average school, No. 2 spends .56 of the average amount. while school No. 5 spends 1.37 times the average amount of 43 cents. Schools Nos. 9 and 7 spend the largest amounts, and schools Nos. 4, 2, 3, the smallest amounts and lie respectively on this score in the upper and lower fourths. The other four schools on the cost for administration lie within the middle half and have been so indicated by the bracket at the top of the chart. From this curve, therefore, it is possible to read from the same chart as that containing the total cost curve, the relative amount spent for administration as compared with the average amount so expended. In each case the "danger zones" and "safety zones" have been indicated.

THE OTHER CURVES

The ratios of geometrical deviation for the costs of instruction, operation, maintenance, and miscellaneous purposes have been com-

puted in the same manner as the corresponding ratio for administration costs and the respective results tabulated in columns 10, 15, 20, and 25 of Table II, and used in locating the "zigzag" curves in parts B, C, and D of the graph. As these curves are interpreted in the same manner as that for administration on Part A, a discussion of them is unnecessary. In case two such curves are placed on the same figure, as in Part D, the middle-half zone for each may be indicated at the top and at the bottom of the chart.

THE RELATIONSHIP OF ANY CURVE TO THE TOTAL COST CURVE

A comparison between the total cost curve and any curve representing a function of expense for any institution gives the proportion going for that function as it is related to the average proportion for all schools combined going for the same purpose. Thus, in Part A, it is seen that the cost of administration in school No. 9 is 3.93 times the average cost of 43 cents, and that the total cost is 3.06 times the average total cost of \$5.97. If the first ratio (3.93) is divided by the second (3.06), the quotient thus obtained (1.28) shows that the proportion going for administration in that institution is 28 per cent higher than the average. Such quotients have been termed geometrical deviations from the total cost ratio and are shown in Table II in paired columns, 6 and 7, 11 and 12, 16 and 17, 21 and 22, and 26 and 27. The first column of each pair contains those deviations which are less than unity, and the second column those which are in excess of unity, and they are called negative and positive deviations respectively. It is seen, therefore, in the diagram (Part A), that the proportion spent for administration is higher than the average in schools Nos. 9, 7, 5, 6, 3, and 8, and lower than the average in schools Nos. 4, 2, and 1. Schools Nos. 5, 1, and 3 spend about the average proportion. If all schools spent the average proportion, viz., 7.2 per cent as shown in Table I. the dotted curve would coincide throughout with the total cost curve. The percentage variations from the heavy continuous curve indicate the magnitude of the deviations from the average practice.

It now becomes necessary to show that the geometrical deviations from the total cost curve express correctly the magnitude of the deviations as measured in the usual way, viz., by computing the proportion of the total cost in each school going for the various functions of expense and comparing the ratios thus obtained with the corresponding average ratio for all schools combined. By referring again to Table II, it will be noted that the \$18.25 spent by school No. 9 is distributed as follows: \$1.69, or 9.3 per cent, for administration; \$10.50, or 57.6 per cent, for instruction; \$2.31, or 12.6 per cent, for operation; \$1.35, or

7.4 per cent, for maintenance; and \$2.40, or 13.1 per cent, for miscellaneous costs. The average percentage spent for administration is 7.2, while the percentage so spent by school No. 9 is 9.3. It is evident that this school spends $\frac{9.3}{7.2}$ times the average. This ratio without question measures the geometrical magnitude of the deviation of this school's expenditures from the general practice. Therefore $r = \frac{9.3}{7.2}$. In determining this particular relationship from the graph we have assumed that $r = \frac{3.93}{3.06}$. If our assumption is correct, the first fraction equals the second, or $\frac{9.3}{7.2} = \frac{3.93}{3.06}$.

If the amounts from which these four relationships were derived in Table II are substituted for them, the verification of the equality is at

once apparent.

$$9.3 = \frac{\$1.69}{\$18.25}, \quad 7.2 = \frac{\$0.43}{\$5.97},$$
$$3.93 = \frac{\$1.69}{\$0.43}, \quad 3.06 = \frac{\$18.25}{\$5.97};$$

or substituting in the equation above,

or
$$\frac{\frac{\$1.69}{\$18.25}}{\frac{\$0.43}{\$5.97}} = \frac{\$1.69}{\frac{\$0.43}{\$18.25}}$$
or
$$\frac{\$1.69 \times \$5.97}{\$18.25 \times \$0.43} = \frac{\$1.69 \times \$5.97}{\$0.43 \times \$18.25}$$

$$1 = 1.$$

Thus it is shown that the ratios used in interpreting the graphs are equal in magnitude to the ratios usually employed in showing such relationships. It is unnecessary, therefore, to compute the percentage distribution of expenditures for each institution or even for all schools combined, as such comparisons are readily apparent from the graphs. For practical purposes it is not necessary to compute the percentages given in Table II, columns 8, 13, 18, 23, and 28. It has now been definitely shown that when the supplementary curves fall to the right of the total cost curve the proportion is higher than the average pro-

portion so expended, and when such curves fall to the left of the total cost curve the proportion is lower than the average proportion.

THE SAFETY ZONE FOR PROPORTIONAL DISTRIBUTION

Conditions may make it undesirable for a school to lower its total cost per student week, but a readjustment of the distribution of costs may be feasible. Thus, it may not be possible for school No. 9 to lower its average cost of \$18.25 per attendance week, since the school is very small, having only 1,558 attendance weeks during the year, and since the cost per student must remain necessarily high. The school also is located in a state where the winter is long and fuel costs are high.

The question, therefore, of a readjustment of expenditures is the only way in which the institution can conveniently conform to the more general practice of normal schools. The question now is: Does the school spend too high a proportion for administration as measured by middle practices? As previously pointed out, it spends 1.28 times the average proportion spent by all nine schools. Does this ratio fall within the "safety zone?" Upon examining column 7, Table II, it is found that three schools have greater percentage deviations than this, viz., in school No. 8, 2.08; in school No. 6, 1.70; and in school No. 7, 1.33. Since there are nine schools in the list, the two having the highest ratios are placed in the upper fourth. These facts have been indicated on the diagram (Part A) by the letter "H" denoting "high." Upon examining column 6 of the same table, it will be noted that schools Nos. 4 and 2 expend the lowest proportions for administration and have been placed accordingly in the first fourth and marked "L" denoting "low." These facts indicate that the proportion spent for administration (1.28) by school No. 9, although considerably higher than the average (1), is still within the "safety zone." The limits of the middle half have been chosen half way between the second and the third highest and between the second and third lowest schools, and have been inserted in Table II, at the bottom of columns 6 and 7. Thus, on administrative costs a school may vary from .81 to 1.51 in its deviation from the total cost curve without being subjected to derogatory criticism. Corresponding actual and permissible deviations have been ascertained for the other functions of expense as shown in the paired columns 11 and 12, 16 and 17, 21 and 22, and 26 and 27 of the same table. For the sake of economy in printing, positive and negative deviations may be placed in one column. In the same manner, as explained above for "administrative" deviations, the "danger zones" for the other functions of expense have been marked "H" (high) and "L" (low).

WHERE TO BEGIN A CHANGE IN THE DISTRIBUTION OF COSTS

By observing all parts of the diagram simultaneously, it will be noted that school No. 9 spends relatively high proportions for administration, for instruction, and for miscellaneous costs, and relatively low proportions for operation and maintenance. Other things being equal, it should first reduce its miscellaneous costs, since on this score it falls decidedly within the upper "danger zone" (10.44). To come within zonal bounds this ratio must be reduced to at least 1.56. place to effect a reduction is in the cost of administration, since this curve (Part A) intersects the line representing school No. 9 at 3.93, while the instruction cost curve (Part B) intersects it at 3.69, indicating that the proportion spent for general control is relatively higher than that spent for teaching purposes. If this school must run at its present cost per student week (\$18.25), it may be necessary to increase the proportion spent for maintenance, as this institution now spends its smallest proportion for this purpose as judged by the average proportion spent for this function by all schools. An inspection of the school plant may reveal the fact that repair and replacement work is badly needed. It may be that janitors are underpaid and that the proportion allotted for operation should be increased.

It can be seen from the graph that school No. 9 has a very high total unit cost (\$18.25)—so high that a justification of it is rather difficult to make. No other school spends so large an amount. Why is it necessary to triple the cost of schooling to secure the advantage of a small institution? For administration this school spends almost four times as much as it costs in the "average" school. Possibly the principal receives a high salary, or perhaps the board of directors is well paid. The curves clearly show where the investigation of school costs should begin, if they have been correctly classified and reported. What causes the unusual expenditures for miscellaneous purposes? On this score only does this school fail to make a defendable distribution of costs, since it is the only point for this school marked "H" or "L." On maintenance costs this institution falls within the "safety zone," spending only 1.22 times the average for all nine schools.

In a similar manner the finances of the other institutions may be analyzed and questioned.

It is the duty of the statistician to lay the facts bare and to point out relationships as has been done in these diagrams; it is the function of an administrator or school surveyor to make suggestions for modifying the practice exposed or to justify the expenditures incurred.

It should be remarked that greater deviation from the general practice usually prevails where the function of expense is small, and

less deviation where the function is large: thus on the cost of instruction it is not the common practice to spend less than .59 or more than 1.74 of the average as indicated by the upper middle-half zone in Part B, while for miscellaneous purposes the permissible range varies from .43 to 2.47 as shown by the lower middle-half zone on Part D The same statement holds for the variation of the proportional amounts so spent. Thus, for instruction, a school may deviate from the total cost ratio only to the extent of .56 and 1.36, while for maintenance costs the corresponding permissible deviations range from .38 to 1.56.

With these explanations clearly in mind, it is seen that a multiplicity of relationships is unfolded in the diagrams. Two further comparisons also may be made. The costs in one school may be compared directly with the costs in another school by disregarding the average costs and middle halves indicated. For instance, school No. 7 spends more than school No. 2 per student week. In the former school, both the absolute amount and the proportional amount spent for administration per attendance week is greater than the corresponding costs in the latter. For the costs of operation of plant, the relative magnitude of these costs is reversed, school No. 2 spending more than school No. 7.

In case it is desirable to compare the costs in a school not represented in the graph with the costs in the nine schools chosen for this study, it is necessary only to ascertain the total cost and the cost for each function of expense per attendance week and to divide these quotients by the average costs for the nine schools represented. The costs in the additional school are then superimposed on the graphs the total cost per attendance week in the school determining its position. Thus, if the total cost per week is found to be \$10.00, a new "school" line would have to be drawn between school No. 7 and school No. 5 on each diagram. An added space can easily be conceived, but it is not necessary to an interpretation of relationships. supplementary curves, representing administration, instruction, etc., would be "broken" between schools Nos. 7 and 5 to lead to and from the new point thus added to each part of the figure. The only disadvantage incident to such an addition is that the costs in the new school do not operate in determining the averages or the middle halves. If a sufficiently large number of instances are incorporated in the original drawings, the modification accompanying the addition of a single new school would not ordinarily materially affect the predetermined standards. In fact, it is often desirable to compare a school system or an institution with some related standard of which it is not a part.